MATH5010 Linear Analysis (2020-21): Homework 4. Deadline: 16 Feb 2021

Important Notice:

♣ The answer paper must be submitted before the deadline.
♠ The answer paper MUST BE sent to the CU Blackboard. Please refer to the course web for details.

1. Let $X := \mathbb{R}^2$ be a two dimensional real vector space and let A be the matrix $\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$.
   Define a mapping $T : X \to X$ by $Tx = Ax$ for $x \in X$. Suppose that X is endowed with the $\|\cdot\|_\infty$-norm, that is $\|x\|_\infty := \max(|x_1|, |x_2|)$ for $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in X$. Find $\|T\|$.

2. Recall that c_{00} denotes the finite sequence space which is equipped with the $\|\cdot\|_\infty$-norm. Let $T : c_{00} \to c_{00}$ be the linear map given by

$$T(x)(k) := kx(k)$$

   for $k = 1, 2...$ and $x \in c_{00}$. Show that T is a discontinuous map.

$* * *$ **End** $* * *$